Chapter 5

Air Impact on Green Sand

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PROBLEM STATEMENT: Toowoomba Foundry is a producer of cast-iron parts for a number of automobile and agricultural machinery manufacturers. To produce their cast-iron products, such as truck brake drums and water pump housings, they use a conventional method involving making a mould, for every item, out of sand. Once made, the mould is filled with molten metal which is then cooled and the mould is then broken to reveal the cast object.

The technology of creating a sand mould consists of placing sand over a pattern and then compressing the sand which has sufficient rigidity that the pattern can be removed easily, and the sand will remain in place when molten metal is poured onto it.

The sand itself has some very special properties and consists of some relatively uniform grains of "Dune" sand mixed with water and very fine particles of bentonite (clay) which cover the sand. The purpose of this project will be to investigate how the sand gets compacted. The method of interest consists of using a sealed box with the pattern as the base in which the sand is loosely placed. The box is then very suddenly subjected to high pressure air (around 7 atmospheres of pressure). If the pressure is applied too slowly very little compaction occurs while when applied very quickly the sand becomes very well compacted with a particularly strongly compacted region adjacent to the pattern. However, the rapid compaction tends to create regions relatively void of sand and hence causes problems when the molten sand is poured in.

Can a model be made to demonstrate how the sand compacts so that methods of creating good compaction without voids can be suggested?

5.1 Introduction

An Australian steel casting company, Toowoomba Foundry, employs a sand mould casting procedure as part of its operation. To produce a mould, a sand-based substance called *green sand* is poured into a box containing the moulding pattern. Subsequently, high pressure air at 0.7 MPa is quickly (on the time scale of 0.1 s) applied to the top of the sand to compress it. The profile of the pressure pulse is shown in Figure 5.1. Upon compression, the sand is packed into a solid structure which preserves the shape of a pattern that is later used for casting.

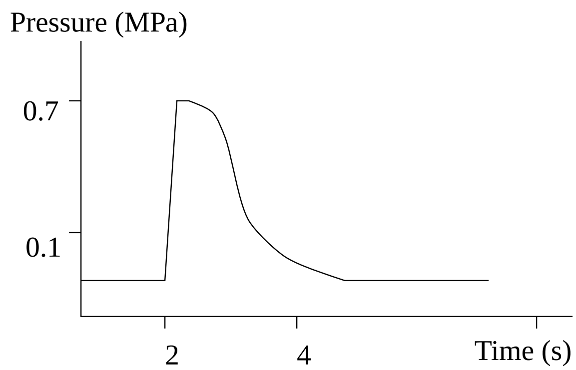

Figure 5.1: *Compaction pressure pulse.*

To optimize the compaction process, the manufacturer wishes to reduce inhomogeneities in the compacted sand that lead to imperfections in the mould. An empirical solution to this problem involves the application of two consecutive compacting air pulses as shown in Figure 5.2.

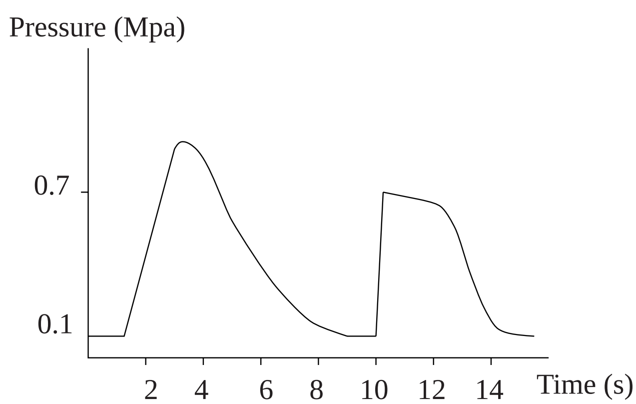

Figure 5.2: *Preceding pressure pulse with a precompaction pulse.*

It has been observed that the mould imperfections, *soft spots*, occur at specific positions in the mould and that the size and location of such imperfections depend on the geometry of the pattern. An example of such a dependency is given in Figure 5.3. Other tools for reducing the size of soft spots at the manufacturer's disposal include: 1) roughening of the pattern surface, 2) the introduction of air vents at the outer surface of the mould.

The object of this research was to develop a qualitative mathematical model of the problem and suggest improvements for the manufacturing process.

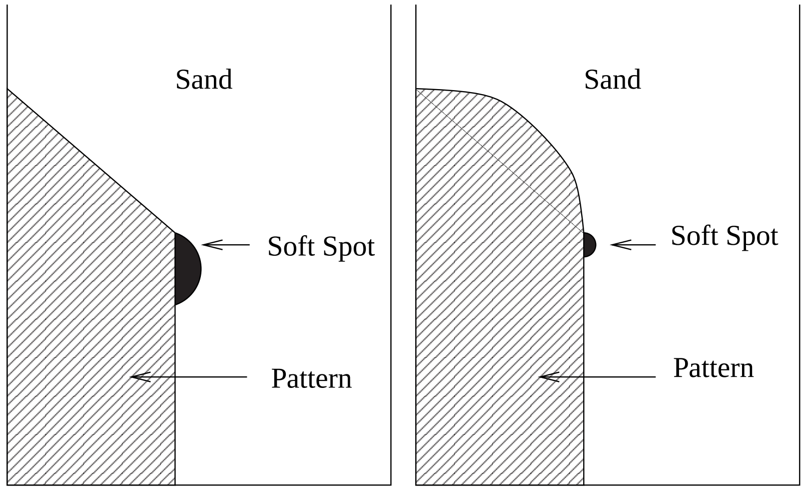

Figure 5.3: *Reduction of soft spots due to the change in mould shape: larger for sharp angles (left figure), smaller for smooth profiles (right figure).*

5.2 Physical properties and assumptions

1. The actual particles of solid in the sand mixture are assumed to be incompressible. However, the air fraction of the sand mixture is compressible. It is the air fraction that makes the sand a compressible material.
2. During the compaction process, the permeability of the sand changes insignificantly. This assumption was made on incomplete experimental data, and should be verified experimentally.
3. Based on experimental data, the sand never experiences decompression.
4. Air is treated as an ideal isothermic Newtonian fluid with constant viscosity.
5. We consider a one-dimensional model in space. The equations are based on the classical conservation laws of mechanics (i.e. the conservation of mass and the conservation of momentum).

5.3 Formal Construction of the Model

Variables

- y = vertical coordinate
- t = time.

Constants

- $h_0 = 0.4$m.
- $K/\mu = 1/\lambda = 10^{-6}$m^{3}s/kg (Darcy's "constant" for green sand).
- $\alpha_0 = 0.6$ (Fraction of air in uncompacted sand mixture).

State Variables

- $\alpha(t, y)$: Volume percentage of air in green sand mixture.
- $\rho_A(t, y)$: Density of air.
- $\rho_S(t, y)$: Density of a sand particle.
- $v_A(t, y)$: Velocity of air.
- $v_S(t, y)$: Velocity of a sand particle.
- $h(t)$: Position of the top of sand mixture.
- $P_0(t)$: Experienced pressure of air at top of the sand box.
- $P_A(\rho_A) = \lambda\rho_A$, where $\lambda = (1\text{atm/kg}\cdot\text{m}^3)$: Air pressure as given by an ideal, isothermal gas.
- $P_S(\alpha)$: Pressure of sand on sand (Experimentally obtained).

5.3.1 Conservation of Mass and Momentum

Mass

$$1.\quad \text{Air:}\quad \frac{\partial(\alpha\rho_A)}{\partial t} + \frac{\partial(\alpha\rho_A v_A)}{\partial y} = 0$$

$$2.\quad \text{Sand:}\quad \frac{\partial((1-\alpha)\rho_S)}{\partial t} + \frac{\partial((1-\alpha)\rho_S v_S)}{\partial y} = 0$$

Momentum

$$3.\quad \text{Air:}\quad \frac{\partial(\alpha\rho_A v_A)}{\partial t} + \frac{\partial(\alpha\rho_A v_A^2)}{\partial y} = -\frac{\partial}{\partial y}P_A - \gamma v_A$$

$$4.\quad \text{Sand:}\quad \frac{\partial}{\partial t}((1-\alpha)\rho_S v_S) + \frac{\partial}{\partial y}((1-\alpha)\rho_S v_S^2) = -\frac{\partial P_S}{\partial x}\frac{\partial \alpha}{\partial y} - \frac{\partial P_A}{\partial \rho_A}\frac{\partial \rho_A}{\partial y}$$

5.3.2 Non-dimensionalization

We scale all variables with respect to a set of characteristic sizes: $P=$ (1 MPa) $\bar{P}$, $\rho_A = \rho_0\bar{\rho}_A$, $h = h_0\bar{h}$, $y = h_0\bar{y}$, $t = t_0\bar{t}$, $v = (h_0/t_0)\bar{v}$, $\rho_S = \rho_S$, $\alpha = \alpha$, $\gamma = \gamma$. Now examine the changes to the above equations.

$$1.\quad \frac{\partial(\alpha\rho_A)}{\partial t} + \frac{\partial(\alpha\rho_A v_A)}{\partial y} = 0$$

$$2.\quad \frac{\partial((1-\alpha))}{\partial t} + \frac{\partial((1-\alpha)v_S)}{\partial y} = 0$$

$$3.\quad \frac{\rho_0 h_0^2}{t_0^2(1\text{MPa})}\left(\frac{\partial}{\partial t}(\alpha\rho_A v_A) + \frac{\partial}{\partial y}(\alpha\rho_A v_A^2)\right) = -\frac{\partial P_A}{\partial y} - \frac{\gamma h_0^2}{t_0(1\text{MPa})}v_A$$

$$4.\quad \frac{\rho_S h_0^2}{t_0^2(1\text{MPa})}\left(\frac{\partial}{\partial t}((1-\alpha)v_S) + \frac{\partial}{\partial y}((1-\alpha)v_S^2)\right) = -\frac{\partial}{\partial y}\left(P_A(\rho_A) + P_S(\alpha)\right).$$

If the momentum of the air is significant, then the proper scaling factor would be

$$\frac{\rho_0 h_0^2}{t_0^2(1\text{MPa})} = 1, \text{ or } t_0 \simeq 0.1\text{ms}.$$

If the momentum of the sand was significant then

$$\frac{\rho_S h_0^2}{t_0^2(1\text{MPa})} = 1, \text{ or } t_0 \simeq 3\text{ms}.$$

If the friction of the air on the sand was significant, then $t_0 \simeq 0.16$s which is comparable to the 0.2s supplied by the machine. Hence, the momentum of sand and/or air does not play any significant role. Therefore, the most significant process is governed by the Darcy law. The initial conditions associated with the momenta, $v_A(0, y) = 0$ and $v_S(0, y) = 0$, are discarded.

5.3.3 Simplified Equations

$$\frac{\partial}{\partial t}(\alpha\rho_A) + \frac{\partial}{\partial y}(v_A\alpha\rho_A) = 0 \tag{5-1}$$

$$\frac{\partial}{\partial t}(1-\alpha) + \frac{\partial}{\partial y}((1-\alpha)v_S) = 0 \tag{5-2}$$

$$\frac{\partial}{\partial y}(P_S(\alpha) + P_A(\rho_A)) = 0 \tag{5-3}$$

$$v_A = -\frac{\partial P_A}{\partial y}. \tag{5-4}$$

The initial value conditions and the boundary conditions become:

$\alpha(0, y) = \alpha_0$	$v_A(t, 0) = 0$ (no vents)
$\rho_A(0, y) = 1$	$v_S(t, 0) = 0$
$h(0) = 1$	$\rho_A(t, 0) = 1$ (vents)
$P_S(\alpha_0) = 0$	$dh/dt = v_S(t, h(t))$
$P_S(\alpha) =$ experimental	$\rho_A(t, h(t)) = P_0(t)$.

Considering the third equation and the last boundary condition, we find

$$P_S(\alpha) + \rho_A = P_0(t)$$

since $P_A = \rho_A$.

5.4 Analysis

The relation

$$P_s(\alpha) + \rho_A = P_0(t)$$

is critical. Through some mathematical consideration (omitted for brevity) we can investigate the compression of the sand mixture in some special cases:

1. Assume that the experienced pressure at the top of the sand box increases much faster than $t_0 \simeq 0.16$s. In this case, no air flows through the sand and the sand compacts uniformly. Vents at the bottom of the box do not play any role.
2. Assume that the experienced pressure at the top of the sand box increases much slower than t_0.
   (a) If there are no air vents at the bottom of the pattern, then no compression occurs.
   (b) If there are vents then compression increases linearly; the bottom will be the most compressed region.

5.5 Conclusions

- *Ignore inertia*: Inertia does not play any role in the compaction process.
- *No shock waves*: Since momentum does not contribute to the compaction process, no waves can be propagated.

5.6 Recommendations

- *No shock waves*: Shock waves should not be considered, even in higher dimensions.
- *Permeability*: Values of permeability are neither easily, nor accurately calculated by theoretical means. Moreover, the value of permeability can critically affect the compression of the sand, especially considering that the critical time t_0 is so close to the machine's capability of raising air pressure. The company is urged to calculate the permeability of the sand mixture for various values of α.
- *Partial fill*: Soft spots are believed to be formed by transverse motion of the sand particles. One method to reduce these defects is to fill the box with sand to the "critical shoulder", followed by a fast compression. Now, fill the box and perform another fast compression.
- *Shoulder air vents*: Place air vents on the "shoulder" above the defect. For this to be effective, the increase in air pressure must occur over a time scale of $\simeq 10t_0$.
- *Plastic top*: If an airtight cover were placed on top of the sand, then the air pressure in the sand-air mixture would be much lower than without the cover. Hence, the sand must compact more. Moreover, as the air pressure in the sand-air mixture is lower, the time designated for pressure release can be shortened significantly.
- *Special air vents*: If air vents are placed at the bottom of the box and are opened shortly after the pressure reaches its maximum, we expect the efficiency of the machine to increase drastically. Installing such vents could lead to two advantages:
  - Denser compaction for the same air pressure pulse.
  - The relaxation time can be reduced.