Dynamic Spectrum Access via Channel-Aware Heterogeneous Multi-Channel Auction With Distributed Learning

Marjan Zandi, Student Member, IEEE, Min Dong, Senior Member, IEEE, and Ali Grami, Senior Member, IEEE

Abstract—We consider the design of dynamic spectrum access (DSA) mechanism. Assuming heterogeneous primary channels with distinct availability statistics unknown to each secondary user (SU), we consider the auction-based approaches for spectrum access. We first apply a unit demand (UD) auction by exploring the instantaneous link condition of each SU for its throughput maximization. To address the disadvantages faced in the UD auction, we propose a learning-based unit demand (LBUD) auction. It incorporates a distributed learning of the primary channel availabilities into the auction mechanism to explore both primary channel availability statistics and instantaneous link gains of the SUs for their throughput maximization. The new mechanism not only substantially reduces communication overhead, but also improves the SUs’ throughputs when the primary channels have dissimilar availability statistics. We show that the proposed LBUD auction for channel allocation among SUs preserves the strong property of dominant strategy incentive compatible.

I. INTRODUCTION

DESIGNING dynamic spectrum access (DSA) mechanisms for efficient utilization of the spectrum is one of the main challenges faced in building cognitive radio networks. A hierarchical cognitive radio network consists of a primary network where primary users are licensed to use the network spectrum, and secondary users (SUs) who opportunistically use the idle channels in the primary network unoccupied by the licensed users. The channel availability statistics of the primary network are typically unknown to the SUs. Through limited spectrum sensing, the SUs search for idle channels and make decisions for channel access. The dynamic access mechanism can be designed in a coordinated, distributed or hybrid fashion. Designing a policy or mechanism for spectrum access among SUs involves key challenges in two aspects: One is on the learning of primary channel conditions by SUs, i.e., how to provide efficient online learning of the primary channel availability statistics based on the sensing observation history. The other is on the handling of access among SUs. The latter not only includes how to provide an effective mechanism to resolve collisions among SUs, but also how to effectively explore the link conditions of the SUs themselves for opportunistic access. All the above issues directly impact the SUs’ throughputs.

Consider a cognitive radio network with independent primary channels and SUs. Distributed design for spectrum access is desirable to reduce communication overhead and/or delay incurred. Several decentralized learning and access policies have been recently developed by formulating the problem as a decentralized multi-armed bandit (MAB) problem, i.e., to select the best out of channels for access in a distributed manner. In these policies, the unknown mean availability of each channel is estimated through a learning process based on each SU’s own sensing observation history. Relying on the estimated mean channel availabilities, each policy designs a different mechanism to avoid or resolve collisions among SUs for their access to the most available primary channels.

Different from MAB formulations, auction design has recently attracted interests for dynamic spectrum access in cognitive radio networks. Treating available channels as objects and SUs as bidders, the channel selection or assignment can be made through an auctioning process. Typically, an auctioneer (or coordinator) is required in such an auction. Compared with the contention-based distributed policies described earlier, it is a collision-free approach for spectrum access among SUs. Several challenges are faced in designing an auction for dynamic access: Each SU needs to decide which primary channels to bid for. Since the number of primary channels can be large, and each channel has different loading statistics, bidding for all the primary channels may be undesirable in terms of both large communication overhead and poor throughput due to selecting an often occupied channel. Then, the question is which channel or a subset of channels each SU should bid for, especially when channel availability statistics are unknown to the SUs. Moreover, each SU’s throughput directly links to its own instantaneous link condition over the selected

primary channel. Although important, such a secondary link condition is typically ignored in the existing distributed policies mentioned above. However, to maximize the SUs’ throughputs, both primary channel load and secondary link condition should be taken into account in the auction process for channel selections. In addition, each SU intends to maximize its own payoff through bidding. Thus, it is desirable for the auction mechanism to possess some nice properties for individual performance guarantee.

A. Contributions

In this paper, we consider an auction-based approach for spectrum access among SUs and try to address the challenges mentioned above in our auction mechanism design. Our design incorporates distributed online learning of the primary channel availability statistics and explores the secondary instantaneous link condition for multi-user multi-channel diversity gain to improve each SU’s throughput. Specifically, we consider primary channels with heterogeneous availability statistics. Through auctioning, each SU obtains a channel and accesses it if idle. Viewing this problem as $M$ bidders bidding for $N$ heterogeneous objects, we adopt a unit demand (UD) auction, known as Demange-Gale-Sotomayor (DGS) auction [2], to determine channel selection of each SU based on its instantaneous rate over each channel.

To address some main disadvantages faced in the UD auction in this problem, we further propose a learning-based unit demand (LBUD) auction. The payoff is designed using each SU’s instantaneous rate over each channel. Each SU only bids for the $M$ most available channels which are learned by the SU distributively using its own sensing history. Note that with unknown primary channel statistics, selecting which set of $M$ channels to bid is highly nontrivial. Our proposed LBUD auction incorporates the design of distributed learning of primary channel availability at each SU. The choice of $M$ channels to bid is determined by the learning of channel availabilities, where the learning process makes a proper trade-off between exploration and exploitation for learning efficiency. Compared with the UD auction, the proposed LBUD auction not only significantly reduces the required communication overhead, but also improves the throughput performance by avoiding selecting channels with low availability. Focusing on the auction itself for channel allocation purpose, we further show that, with the payoff considered, the LBUD auction preserves the strong property of the DGS auction, i.e., it is dominant strategy incentive compatible (DSIC) (the definition is given in Section IV-B). This means that, each SU will achieve its maximum payoff by bidding truthfully\(^1\) using SU’s own instantaneous rate, regardless of whether other SUs bid truthfully or not.

For both UD and LBUD auctions, an iterative procedure is used to determine the channel assignment to each SU. To improve the convergence speed of the iterative procedure, instead of fixed price increment, we further propose an adaptive price increment algorithm to determine price increment in each iteration. Simulations show the effectiveness of our proposed auction mechanism in throughput gain over other existing policies by bidding only selective primary channels and exploring instantaneous secondary link conditions.

To the best of our knowledge, we are the first to apply the DGS auction designed for bidding heterogeneous objects with unit demand to the distributed dynamic access design. Furthermore, our proposed LBUD auction incorporates distributed learning of the primary channels into the auction mechanism to explore both channel availability statistics and instantaneous link gain of SUs, in order to maximize SUs’ throughputs. Such a joint consideration of both primary channel and secondary link is not considered in either existing decentralized MAB policies or auction-based access mechanisms.

B. Related Works

As mentioned earlier, using a decentralized MAB formulation, a few decentralized learning and access policies were proposed [3]–[8]. These existing access policies make channel selection and access solely based on the estimated mean availabilities of primary channels from sensing history, but do not explore the instantaneous fade conditions in the secondary user transmission links.

Game-theoretic approaches have been considered for designing channel selection and access policies in cognitive radio networks [17]–[21], where SUs’ accesses have been modeled and formulated using certain type of games. Again, a joint consideration of primary channel availability statistics and instantaneous link gains of the SUs is not considered in the model and game formulation of these works.

Auction-based approaches have recently attracted many research interests for efficient spectrum access, sharing, or leasing [10]–[16], [22], [23]. Many of these works treat primary channels as multiple objects for SUs to bid, and allow each SU to be assigned multiple channels to maximize certain defined utility [10], [13]–[16], [23], instead of requesting only one channel (unit demand). In [13], spectrum trading in TV band is considered by taking into effect of imperfect spectrum sensing. To handle bidding for multiple objects, a multi-unit sequential sealed-bid first-price auction is proposed. Spectrum auction with multiple primary spectrum auctioneers is considered in [14], and a progressive auction is proposed for each SU to select its best spectrum auctioneer for bidding. In [15] an auction-based cooperative sensing protocol for SUs is proposed. In [16], VCG-based auction mechanisms were proposed for joint interference control and spectrum auction for SUs with mobility. In [10], based on the second-price auction [24], a repeated auction is considered to determine the assignment of available channels to SUs based on certain cost utility function. Bertsekas auction algorithm [25] was proposed as a fast-converging algorithm for the assignment problem. It can be applied to the channel assignment among SUs. To reduce the high communication overhead incurred in the Bertsekas auction algorithm, [22] modified the Bertsekas algorithm and proposed a fully distributed auctioneer-free auction algorithm by using an opportunistic carrier sense.

\(^1\)In truthful bidding, each SU’s bid should reflect its valuation truthfully. In other words, its bid equals to its valuation.
multiple access (CSMA) assignment scheme. Besides the above mentioned single-side auctions focusing on bidding among SUs, treating both secondary and primary users as bidders for channel access, double-sided auction is considered in [11], [12].

In all the above works, either the primary channels for auction are assumed available or they are assumed homogeneous in nature without taking into account the different loading conditions (i.e., availability statistics) and their impact on the channel assignments. In addition, different from our problem, each SU is allowed to win multiple channels depending on the auction outcome, instead of each SU selecting one channel to access. Furthermore, the second-price (Vickery) auction [24] or the Vickery-Clarke-Groves (VCG) auction [26] adopted in the existing works are designed for bidding a single object and multiple objects, respectively.

For auction design in economics, the second-price auction was proposed for bidding a single object,2 and was shown to be DSIC. It also achieves the minimum price equilibrium. This auction has then been generalized to the VCG auction for the scenario with multiple objects. For bidding multiple heterogeneous objects with unit demand, the DGS auction was proposed [2], which was shown to be DSIC. Like the VCG auction, the DGS auction also preserves some interesting properties of the second-price auction, i.e., it is DSIC and achieves the minimum price equilibrium. In addition to these common properties, the DGS auction has extra nice properties which motivated us for considering this auction in our work. In practice, bidders may have limited budgets for auction. It is shown in [27] that addressing budgets properly breaks down the incentive compatibility of the VCG auction, while in [28] it is shown that the DGS auction is incentive compatible even if the bidders have budget constraints. In addition, the DGS auction mechanism is group strategy-proof, while the VCG auction is vulnerable to collusion [29], [30]. Due to these properties for the DGS auction, in our work, we consider the DGS auction for bidding the primary channels with heterogeneous channel availability statistics.

C. Organization and Notations

The remainder of this paper is organized as follows. In Section II, we present the network model. In Section III, we apply the UD auction for spectrum access to determine the channel assignment among SUs. In Section IV, to overcome some main disadvantages faced in the UD auction, we propose the LBUD auction mechanism, and study its properties. Furthermore, we propose an adaptive algorithm to improve the convergence rate of the iterative procedure in the auction. Numerical results are presented in Section V. Finally, we conclude in Section VI.

Notations: The main symbols used in this paper are summarized in Table I.

| M       | number of secondary users          |
| N       | number of channels                 |
| n       | current time slot                  |
| X_i(n)  | availability state of the primary channel i at time slot n |
| \theta_i | mean availability of channel i     |
| \hat{\theta}_i | estimated mean availability of channel i by SU j |
| T_i^j(n) | number of times that the SU j senses channel i up to time slot n |
| T_i^j(n) | set of time slots up to the current time slot n that SU j has been the sole user of channel i |
| R(n)    | expected throughput for the secondary network |
| m^j_i(n) | bid of SU j for channel i |
| m^j_j(n) | confidential bidding vector for all the primary channels by SU j |
| m^j_j(n) | bidding vectors of SU j’s opponents |
| A^j_i(n) | allocation of channel i to SU j at time slot n |
| P_i^*(n) | price for channel i at time slot n at iteration l |
| R_i^*(n) | instantaneous rate of SU j on channel i |
| D^j((P_i^j(n))) | demand set for each SU j at time slot n at iteration l |
| B(D^j((P_i^j(n))) | demanders of D^j((P_i^j(n)) |
| B^k(D^j((P_i^j(n))) | exclusive demanders for D^j((P_i^j(n)) |
| \Delta P_{n,l} | price increment at iteration l and time slot n |
| D_{\text{max}}(P_i^j(n)) | minimal overdemanded set |
| F(n)    | index vector for SU j based on its own sensing observation history |
| I_i^j(n) | rank of channel i at time slot n for SU j |
| C_i^j_M(n) | estimated M-best channels of SU j at time slot n |
| S(n)    | average payoff per SU at time slot n |

II. NETWORK MODEL

We consider a slotted primary network consisting of N orthogonal radio channels available to the licensed primary users. A secondary network with M secondary users search and compete for the instantaneous idle channels among these N channels.3 Denote X_i(n) \in \{0, 1\} as the availability state of the primary channel i at slot n, with X_i(n) = 1 as the channel i being available, and it is 0 otherwise. We assume the channel i’s availability state X_i(n) evolves as an i.i.d. Bernoulli random process over time, with X_i(n) \sim \text{Bernoulli}(\theta_i), where \theta_i = \text{E}[X_i(n)], \forall n. Let \theta = [\theta_1, \ldots, \theta_N]^T denote the channel availability vector. We assume elements in \theta are all distinct and unknown to the SUs. Due to hardware limitation, at the beginning of each time slot n, we assume each SU j can only select one channel to sense and, if available, to access. We assume perfect channel sensing is performed.

Let h_i^j(n) be the channel gain between SU j and its (secondary) destination (e.g., base station) over radio channel i.

---

2 In second-price auction, the bidder with the highest bid is winner of the auction and the amount it has to pay is equal to the second highest bid.

3 We assume the secondary network is relatively small in range where SUs may interfere with each other, and SUs see the same primary network activities.
at time slot $n$.\(^4\) Assuming channel $i$ is available for SU $j$ to access, the corresponding instantaneous achievable rate of SU $j$ is $R'_i(n) = \log(1 + P_j h'_i(n)^2/\sigma^2)$ with $P_j$ and $\sigma^2$ being SU $j$’s transmit power and its receiver noise variance, respectively. We assume perfect knowledge of $R'_i(n), \forall i$, at each SU $j$.\(^5\) The expected throughput for the secondary network, under a given access mechanism, is given by

$$R(n) = \frac{1}{n} \sum_{j=1}^{M} \sum_{i=1}^{N} E \left[ \sum_{k \in I_j(n)} X_i(k) R'_i(k) \right]$$

$$= \frac{1}{n} \sum_{j=1}^{M} \sum_{i=1}^{N} \theta_i E \left[ \sum_{k \in I_j(n)} R'_i(k) \right]$$

where $I_j(n)$ denotes the set of time slots up to the current time slot $n$ that SU $j$ has been the sole user\(^6\) of channel $i$. We see that the throughput in (1) is determined by two factors, the primary channel availability and the instantaneous rate of each SU. Our problem is to design a distributed online learning and access mechanism among SUs to maximize the secondary network throughput. Different from existing learning and access designs, our design explores both the instantaneous channel gain over secondary links and primary channel availability to maximize the secondary network throughput.

III. DYNAMIC ACCESS VIA MULTI-CHANNEL AUCTION

A major challenge faced in distributed access among SUs to the primary network is the design of collision avoidance among SUs for their access. We consider an auction-based access mechanism, in which each SU performs online learning of the primary channels individually while their access channel selection is managed by an auctioneer (or coordinator). The benefit of such an auction mechanism is to enable a collision-free access.

Let $S$ denote the set of SUs and $C$ the set of the primary channels. Consider SUs as the bidders and the primary channels as the objects of the auction. In this section, we first consider each SU can bid for any channel in $C$. In Section IV, we modify our auction mechanism to consider bidding channels in a subset of $C$. At the beginning of time slot $n$, SU $j$ sends to the auctioneer a confidential bidding vector of all the primary channels, denoted as $m(j(n) = [m'_1(n), \ldots, m'_M(n)]^T$, where $m'_i(n)$ denotes the bid of SU $j$ for channel $i$. If SU $j$ decides not to participate in bidding of channel $i$, then $m'_i(n) = 0$. We also define $m^{-i}(n)$ as the bidding vectors of SU $j$’s opponents, i.e., $m^{-i}(n) = [m'_k(n)|k \in S/j]$. Based on the bids from SUs, the auctioneer will allocate a channel to each SU.

Note that since the mean availability statistics $\theta_i$’s are distinct, these primary channels that the SUs bid for are considered as the heterogeneous type. Thus, the problem is essentially the bidding of multiple heterogenous objects with unit demand. This is considered as a unit demand auction. In economics, DGS auction [2] was first proposed to handle such a scenario. It is a generalization of the second-price auction [26] which deals with multiple bidders bidding for a single object. The DGS auction preserves some nice properties of the second-price auction. It is a weakly dominant strategy\(^7\) which leads to a dominant strategy equilibrium in which the payoff of each bidder is maximized regardless of other bidders’ strategies. Note that the dominant strategy equilibrium is a Nash equilibrium, but not vice versa. In addition, the DGS auction reaches the minimum price equilibrium [2].

Let $A_i^j(n) \in [0,1]$ indicates the allocation of channel $i$ to SU $j$ at time slot $n$, with the value of 1 if channel $i$ is assigned to SU $j$. Each channel can be assigned to at most one SU, and each SU can be assigned at most one channel. Denote $A_i(n)$ the allocation of channel $i$ to SU $j$ at time slot $n$, and $\mathbf{A}(n)$ the channel allocation for SU $j$ at time slot $n$, respectively. They are given by

$$A_i(n) \triangleq \left\{ A_i^1(n), \ldots, A_i^M(n) : \sum_{j=1}^{M} A_i^j(n) \leq 1 \right\}$$

$$\mathbf{A}(n) \triangleq \left\{ A_1^1(n), \ldots, A_N^N(n) : \sum_{i=1}^{N} A_i^j(n) \leq 1 \right\}$$

In addition, if an SU does not bid for a channel, it will not be assigned to that channel, i.e., $A_i^j(n) = m'_i(n) = 0$. A reservation price $P_{\text{min}}^i$ is given to each channel $i$, indicating the minimum price the auctioneer accepts for a specific channel. Let $P_{\text{min}} = [P_{\text{min}}^1, \ldots, P_{\text{min}}^M]^T$. The auction mechanism uses an iterative procedure to determine the channel assignment for each SU. Let $P_{\text{d}}^j(n)$ denote the price for channel $i$ at time slot $n$ at iteration $l$. Let $P_{\text{d}}^j(n) \triangleq [P_{\text{d}}^1(n), \ldots, P_{\text{d}}^M(n)]^T$.

There are three terms used in the UD auction, demand set, overdemanded set and minimal overdemanded set. We first provide their definitions below.

**Definition 1 ([22]):** Demand set—Demand set for each SU $j$, denoted as $\mathcal{D}(P^j(n))$, is defined as the set of channels that give the current maximal payoff for SU $j$, i.e.,

$$\mathcal{D}(P^j(n)) = \left\{ \arg \max_{i \in \mathbb{C}} R'_i(n) - P_{\text{d}}^i(n) \right\}$$

**Definition 2 ([22]):** Overdemanded set—Define the set of SUs as demanders of $\mathcal{D}(P^j(n))$ as

$$\mathcal{B} \left( \mathcal{D}(P^j(n)) \right) = \left\{ k : \mathcal{D}^k (P^j(n)) \cap \mathcal{D}^k (P^j(n)) \neq \emptyset, \forall k \in \mathbb{S} \right\}$$

\(^4\)Note that channel $i$ indicates the frequency channel SU $j$ occupies, while $h'_i(n)$ is the channel gain over the link between the secondary transceiver, which can be measured by SU $j$.

\(^5\)There may be different ways for each SU to obtain its rate in practice. Without causing interference to primary users, an SU may use the (delayed) rate information obtained from the most recent channel access. Note that, if SUs are bidding only among a subgroup of channels, as what we will propose later in the paper, the delay may be short. In our work, we idealize it to be instantaneous rate. Other short channel probing design for SUs may be possible at the beginning of each time slot to obtain a coarse estimate of its rate, provided the interference to the primary users is kept below a tolerable level.

\(^6\)In other words, $I_j(n)$ only contains the set of time slots up to the current time slot $n$ that SU $j$ does not collide with other SUs on channel $i$.
Define the set of SUs as exclusive demanders for $D_i(P(n))$ as
\[
\mathcal{B}^E(D_i(P(n))) = \{k : D_i(P(n)) \subseteq D_j(P(n)), \forall k \in S\}.
\]
We call the set (of channels) $D_j(P(n))$ being overdemanded at price $P(n)$ if\(^8\)
\[
D_j(P(n)) \subset C \text{ and } |\mathcal{B}^E(D_j(P(n)))| > |D_j(P(n))|.
\]
(6)
In other words, the set of channels is overdemanded, if there are more SUs, whose highest payoff channels are all in this set, than the number of channels in the set.

**Definition 3 ([2]): Minimal Overdemanded set—**An overdemanded set $D \in \mathcal{O}$ is called a minimal overdemanded set if $D \not\subset C, \forall D' \subset D$.

We summarize the UD auction mechanism for the allocation decision:

1) The auctioneer initializes the price to the reservation price for each channel: $P^0(n) = P_{\text{min}}$;
2) For each SU $j, j \in S$, it observes its current valuation of each channel, i.e., the instantaneous rate $R_i(n), \forall i$. Since bidding truthfully is a weakly dominant strategy in the UD auction, we set the bid to be the valuation of the channel, $m'_j(n) = R_j(n)$. SU $j$ then sends $m'_j(n)$ to the auctioneer;
3) The auctioneer obtains the demand set for each SU $j$, $D(n)$, as in (4).
4) Let $D(P(n)) \triangleq \{D_1(P(n)), \ldots, D_M(P(n))\}$. The auctioneer follows an iterative procedure to check whether there is any overdemanded set among the demand sets in $D(P(n))$:
   4.1) If there is no overdemanded set of channels: The channel allocated to SU $j$ is given as $A_j^f(n) = 1$, where $j \in D_i(P(n))$ is selected randomly for $|D_i(P(n))| > 1$.\(^9\) and $A_j^f(n) = 0$, for $j - i \in C \setminus [i]$. The allocation process is completed and terminated. The final price on channel $i$, defined by $P_i(n)$, is given by
   \[
P_i(n) = P_i^{l+1}(n).
\]
   4.2) If there are overdemanded sets of channels:
   4.2a) Let $S^o$ be the set of SUs whose $D_i(P(n))$ is an overdemanded set. The auctioneer collects all the overdemanded sets into a set $O$
   \[
   O = \{D_i(P(n)), \forall j \in S^o\}.
   \]
   4.2b) The auctioneer finds a minimal overdemanded set $D_{\text{min}}(P(n))$ from $O$,\(^10\) and updates the price vector $P_i(n)$, for $i \in D_{\text{min}}(P(n))$:
   \[
P_i^{l+1}(n) = P_i^l(n) + \Delta P_{n,i}
   \]
   where $\Delta P_{n,i}$ is the price increment at iteration $l$ and time slot $n$. Return to Step 3.

**Remark 1:** The minimal overdemanded set might not be unique. Therefore, if there is more than one overdemanded set, then one of them is randomly selected by the auctioneer.

**Remark 2:** It has been shown that the above UD auction mechanism leads to a minimal price equilibrium.\([2]\). That is, let $P^*(n)$ be the price obtained at the end of the auction, and $q(n)$ be any other competitive price vector at time slot $n$. Then, $P^*(n) \leq q(n)$. In addition, since the auction is shown to be weakly dominant strategy, each SU obtains its maximal payoff regardless of other bidders’ strategies.

**Remark 3:** In the above UD auction mechanism, each SU uses its instantaneous rate (of the secondary link) to bid; thus, the channel assignment from the auction depends on the instantaneous link condition for each SU on the primary channels. Therefore, the channel assignment is opportunistic, and the resulting throughput at the secondary network gains from such an opportunistic allocation, in addition to be collision free.

**IV. Multi-Channel Auction via Distributed Learning**

In the UD auction mechanism described in the previous section, each SU bids for all $N$ channels. We are interested in the scenario where $M < N$. This scenario arises in broadband spectrum access where there are a large number of primary channels for consideration.\(^11\) In this case, there are two drawbacks to this approach: First, there are a total of $MN$ bids submitted to the auctioneer which incurs a large communication overhead. Second, the payoff used in determining the channel allocation only reflects each SU’s instantaneous rate of its link on a channel, but does not take into account the different mean channel availabilities among the primary channels. The latter could result in more throughput loss by selecting a primary channel that is less available. To overcome these drawbacks, we propose an adaptive auction mechanism, named the LBUD auction. In this auction, each SU $j$ will adaptively choose the best $M$ channels to bid.

Since the channel mean availability $\theta_i$ of each channel $i$ is unknown to the SUs, to determine the most available $M$ channels, each SU learns $\theta$ distributively from its own sensing outcome and history over time.\(^12\) Let $T_i^j(n)$ denote the number of times that the SU $j$ senses channel $i$ up to time slot $n$. For SU $j$ selecting channel $i$ to sense at time slot $n$, it records the value of $X_i(n)$ as $X_i^j(T_i^j(n))$. For SU $j$, its sensing observation history of channel $i$ up to time slot $n$ is denoted by $X_i^j(n) = [X_i^j(1), \ldots, X_i^j(T_i^j(n))]^T$. SU $j$ estimates $\theta_i$ of channel

\[^{11}\text{This scenario is particularly suitable for the cognitive radio network where the SUs may not be restricted to a certain frequency band and can search among a large set of channels.}\]

\[^{12}\text{Since one of the main motivations is to reduce the overhead, we consider distributed learning of $M$-best channels at each SU. A centralized learning would require each SU to submit its sensing results to the auctioneer and obtain the estimated $M$-best channels from the auctioneer every time slot.}\]
\[ i \text{ at time slot } n \text{ using the sample mean of the observations from } X^i_j(n) \text{ as} \]
\[ \hat{\theta}^i_j(T^i_j(n)) = \frac{1}{T^i_j(n)} \sum_{k=1}^{T^i_j(n)} X^i_j(k). \quad (9) \]

The online learning algorithm, upper-confidence-bound I (UCB1) [31], is a sample-mean based index policy to learn and access \( N \) channels in a single user scenario. Through efficient exploration-exploitation trade-off, the UCB1 algorithm has been shown to be order-optimal in terms of the learning rate over time. Existing decentralized policies [3]–[5] have extended the UCB1 algorithm to a multi-user scenario with decentralized learning at each SU. In the UCB1 algorithm, each SU \( j \) ranks channel \( i \) at time slot \( n \) based on an index, \( I^i_j(n) \), defined as
\[ I^i_j(n) = \hat{\theta}^i_j(T^i_j(n)) + \sqrt{\frac{2 \log n}{T^i_j(n)}}. \quad (10) \]

The SU computes the index vector \( I(n) \equiv [I^1_j(n), \ldots, I^n_j(n)]^T \) based on its own sensing observation history. Note that the two terms in (10) capture the exploration and exploitation trade-off in learning. The sample mean for estimated channel availability in the first term corresponds to exploitation, while the second term is used for exploration which adds weights to those channels that are not sensed often. Thus the trade-off is between choosing a channel with a high estimated availability for immediate throughput maximization and choosing another channel to obtain an improved estimate of its availability.

Define the \( M \)-best channels as those channels whose \( \theta \)'s are among the \( M \) highest ones. We also define the estimated \( M \)-best channels by SU \( j \) as those channels whose indexes \( I^i_j(n) \)'s in (10) by SU \( j \) are among the top \( M \)-ranked. Thus, the estimated \( M \)-best channels reflects the exploration-exploitation trade-off captured by \( I^i_j(n) \) in (10) in the underlying learning. Let \( C^i_M(n) \) denote the set of indexes of the estimated \( M \)-best channels for SU \( j \) at time slot \( n \), i.e.,
\[ C^i_M(n) = \left\{ i : I^i_j(n) \in \left[ I^{i_{(1)}}(n), \ldots, I^{i_{(M)}}(n) \right] \right\}. \quad (11) \]

where \( \{I^{i_{(k)}}(\cdot)\} \) is the ordered statistics of \( \{I^i_j(\cdot)\} \) with \( I^{i_{(1)}}(\cdot) > \cdots > I^{i_{(N)}}(\cdot) \). At each time slot \( n \), SU \( j \) updates its estimated \( M \)-best channel set \( C^i_M(n) \), and form the bidding vector for these channels: \( \hat{m}^i_j(n) = [m^1_k_i(n), \ldots, m^n_k_m(n)]^T \), where \( k_i \in C^i_M(n) \). The auctioneer performs an auction-based allocation using these bidding vectors from SUs. The demand set \( D^i_M(\hat{P}^i(n)) \) for SU \( j \) in this case is given by
\[ D^i_M(\hat{P}^i(n)) = \left\{ \arg\max_{i \in C^i_M(n)} \left( m^i_{\hat{P}^i}(n) - P^i(n) \right) \right\}. \quad (12) \]

We will show in Section IV-B that using the true valuation of a channel as the bid, i.e., the instantaneous rate on the channel, will lead to the maximum payoffs among SUs. Therefore, we set \( m^i_{\hat{P}^i}(n) = R^i(n) \) in the following. Using \( D^i_M(\hat{P}) \), we design an iterative procedure to determine the channel allocation among \( M \) SUs. The LBUD auction mechanism is described in Algorithm 1.

**Algorithm 1 Learning-Based Unit Demand (LBUD) Auction**

1. **Input:**
   - \( n \): Current time slot
   - \( l \): Current iteration

2. **Init:** Set the price vector \( \hat{P}^0(n) \leftarrow P_{\min} \)

3. **SU** \( j \) updates \( \hat{\theta}^i_j(n) \) using (9), \( \forall i, \) and obtain \( C^i_M(n) \) using (11).

4. **SU** \( j \) observes its current link rate \( R^i_j(n) \), \( \forall i \) and sends a confidential bidding vector \( m^i_j(n) \).

5. The auctioneer obtains the demand set \( D^i_M(\hat{P}^i(n)) \) for SU \( j \) using (12).

6. The auctioneer checks whether there is any overdemanded sets among the demand sets \( D^i_M(\hat{P}^i(n)) \):
   a) The auctioneer obtains the exclusive demanders \( B^M(D^i_M(\hat{P}^i(n))) \) of \( D^i_M(\hat{P}^i(n)) \) as in (5), \( \forall j \).
   b) The auctioneer checks whether \( D^i_M(\hat{P}^i(n)) \) is an overdemanded set or not as in (6), \( \forall j \):
      - **If there is no overdemanded set:** The channel allocated to SU \( j \) is given as \( A^i_j(n) = 1 \), where \( i \in D^i_M(\hat{P}^i(n)) \) is selected randomly for \( |D^i_M(\hat{P}^i(n))| > 1 \) and \( A^i_j(n) = 0 \), for \( i^* \in C^i_M(n) \). The allocation process is completed and terminated. The final price on channel \( i \), \( P_i(n) \), is given by \( P_i(n) = P^i_{l+1}(n) \).
      - **If there are overdemanded sets:**
         i) The auctioneer forms set \( \mathcal{O} \) as in (7).
         ii) The auctioneer finds a minimal overdemanded set \( D^i_{\min}(\hat{P}^i(n)) \) from \( \mathcal{O} \), and updates the price vector \( \hat{P}^i(n) \), for \( i \in D^i_{\min}(\hat{P}^i(n)) \), as
            \[ P^i_{l+1}(n) = P^i_l(n) + \Delta P_{n,j}. \quad (13) \]
         iii) Set iteration \( l \leftarrow l + 1 \); return to Step 5.

Note that, in terms of overhead, in the LBUD auction, each SU only submits \( M \) bids along with the channel index set \( C^i_M \). The total overhead is \( M^2 \) bids plus \( M \log_2 M \) bits, in contrast to \( MN \) bids in the UD auction. For accessing broadband spectrum with \( N \gg M \), the reduction in communication overhead is significant. Performance-wise, each SU only bids among its estimated \( M \)-best channels, and at the same time, the channel allocation is based on the instantaneous (secondary) link condition on each channel. Thus, the LBUD auction is designed to 1) ensure good channel selection in the mean sense; 2) enjoy the gain from opportunistic channel selection. Note that existing distributed learning and access policies [3]–[5] for spectrum access only ensure good channel selection in the mean sense without considering instantaneous channel condition of SUs’ communication links.

For the LBUD auction, the winning channel will be selected only among those highly available channels. When the primary channel mean availability values in \( \theta \) are relatively spread, this
will result in the LBUD auction outperforming the UD auction. This is because bidding only among the \( M \)-best channels avoids SUs to access the channels which are less likely to be available (as in the UD auction), even though the SU’s instantaneous rates over these channels are high. However, when the values in \( \theta \) are close, the UD auction may outperform the LBUD auction, due to the multi-channel diversity gain from the opportunistic selection of \( M \) out of \( N \) channels. To cover a broad range of the distribution of primary channel availability statistics \( \theta \), in practice, we should consider both mechanisms to adapt to different types of traffic conditions over channels in the primary network. It should be mentioned that the LBUD auction becomes the UD auction when \( M \geq N \). However, for \( M < N \), there is a nontrivial learning process of the \( M \)-best channels involved. In this case, the LBUD auction improves the throughput performance and reduces the communication overhead than the UD auction.

A. Adaptive Algorithm for Price Increment \( \Delta P_{n,i} \)

The iterative procedure in both the UD and the LBUD auctions requires the price update with the price increment \( \Delta P_{n,i} \), as shown in (8) and (13). Setting the appropriate price increment \( \Delta P_{n,i} \) is important as it directly affects the convergence behavior of the iterative procedure for the auction. If the increment is too small, the overdemanded set will not change over several iterations, resulting in slow convergence. However, if the increment is too large, the iterative procedure may not guarantee to converge. Note that the DGS auction is originally proposed for integer valuations and prices. Due to this, the price increment in the iteration procedure is fixed to \( \Delta P_{n,i} = 1 \), i.e., the minimum possible difference of two non-identical valuations. The convergence has been shown with this unit increment [2]. In our case, the valuations (instantaneous rates) are real numbers. In this case, we can similarly set \( \Delta P_{n,i} \) to be the minimum difference of instantaneous rates of all channels that each SU bids for.

Let \( \tilde{C}(n) \) be the primary channel set considered by SU \( j \): for DGS, \( \tilde{C}(n) = C \), and for LBUD, \( \tilde{C}(n) = C_m(n) \). Then, the price increment \( \Delta P_{n,i} \) is set as

\[
\Delta P_{n,i} = \min \left\{ |R_k(n) - P_m(n)| : i, m \in \tilde{C}(n), i \neq m, \forall j \right\}
\]

(14)

where \( \Delta P_{n,i}^b \) is denoted as the baseline price adjustment.

Note that, by this baseline price increment method, \( \Delta P_{n,i} \) is fixed for each auction procedure, but varies from slot to slot. Using the price increment suggested in (14) may lead to slow convergence. To improve the convergence rate, we propose an adaptive algorithm which updates \( \Delta P_{n,i} \) adaptively in each iteration \( l \). It is described as follows (for the auctioneer, at time slot \( n \) and iteration \( l \)).

Let \( P_i(n, l) \) denote the current payoff of SU \( j \) over channel \( i \) at time slot \( n \) at iteration \( l \), i.e.,

\[
P_i(n, l) = R_i(n) - P_i(n).
\]

(15)

The auctioneer obtains \( P_i(n, l) \) for each SU \( j \in S \) and channel \( i \). Let \( k'_1 \) and \( k'_2 \) denote the channels with the current maximum and second maximum payoff for SU \( j \) at time slot \( n \) and iteration \( l \), respectively. They are obtained by

\[
k'_1 = \arg \max_{i \in \tilde{C}(n)} \rho'_i(n, l) \quad (16)
\]

\[
k'_2 = \arg \max_{i \in \tilde{C}(n)} \left\{ \rho'_i(n, l) : \rho'_i(n, l) \neq \rho'_j(n, l), i \neq j \right\} \quad (17)
\]

(For each SU \( j \), let \( \Delta \rho(n, l) \) be the payoff difference over the two channels \( k'_1 \) and \( k'_2 \), given by

\[
\Delta \rho(n, l) = |\rho'_{k'_1}(n, l) - \rho'_{k'_2}(n, l)|. \quad (18)
\]

The auctioneer updates the price increment \( \Delta P_{n,i} \) as

\[
\Delta P_{n,i} = \min_{1 \leq j \leq M} \Delta \rho(n, l). \quad (19)
\]

The convergence of the iterative procedure with the above adaptive price increment is shown in the following proposition.

**Proposition 1:** Under the proposed adaptive price increment algorithm, the iterative procedure in both the UD and the LBUD auctions converges to the same channel assignment outcome as that of the baseline price adjustment method.

**Proof:** See Appendix A.

**Remark 1:** It can be seen that the price increment \( \Delta \rho(n, l) \) is adaptively determined based on the gap of current top payoffs \( \rho'_i(n, l) \) over primary channels among SUs. Intuitively, the adaptive price increment algorithm is designed to skip those iterations where there is no change to the overdemanded sets and minimal overdemanded sets among SUs, and thus they will not affect the outcome of channel assignment result. By doing so, the algorithm avoids those “null” iterations resulting in fewer iterations to reach the same channel assignment solution.

**Remark 2:** As indicated in Proposition 1, the adaptive price increment results in the same allocation as that of the baseline price increment. Therefore, our proposed adaptive price increment algorithm does not affect the allocation efficiency of the auction mechanism (UD or LBUD).

We summarize the proposed adaptive price increment procedure in Algorithm 2. Such an adaptive increment avoids unnecessary iterations and expedites the convergence to the final channel assignment for each SU at the auctioneer. In Section V, through simulations, we show that the proposed adaptive price increment algorithm substantially improves the convergence rates of both the DGS and the LBUD algorithms.

**Algorithm 2** Adaptive Price Increment Algorithm (\( \Delta P_{n,i} \) at the \( l \)th iteration)

1. **Input:**
   - \( n \): Current time slot
   - \( l \): Current iteration
2. Obtain \( P_i(n, l) \) as in (15) for each SU \( j \in S \) and channel \( i \).
3. Obtain \( k'_1 \) as in (16) and \( k'_2 \) as in (17).
4. Obtain \( \Delta \rho(n, l) \) as in (18).
5. Update the price increment \( \Delta P_{n,i} \) as in (19).

13 If there are multiple channels having the same maximum (or second maximum) payoffs, randomly select one channel as \( m' \) (or \( m'' \)).
B. Property of the LBUD Auction and Discussions

An auction mechanism is said to be incentive compatible if all bidders will receive the maximum payoffs when their bids reflect their valuations truthfully [32]. Furthermore, a strategy is called dominant strategy incentive compatible, i.e., DSIC, where each bidder achieves its maximum payoff by bidding truthfully irrespective of whether the other bidders bid truthfully or not [32]. Under a dominant strategy, the bidders of this auction do not need to collect or analyze any information about the status or intentions of their competing bidders.

Bidding truthfully simplifies decision making of the SUs in an auction. It is because by bidding truthfully, the SUs need to know only their own valuations and therefore, they do not depend on knowledge of the other bidders and their distribution of possible values. It has been shown in [2] that bidding truthfully is a dominant strategy in the UD auction. Thus, if we consider the auction process itself for channel allocation purpose, with the payoff defined in (4), the UD auction described in Section III is DSIC. Under the same consideration, we now show that the same property also holds for the proposed LBUD auction as the underlying learning of primary channels by each SU improves.

Proposition 2: Consider the auction process for channel allocation of M-best channels with payoff defined in the objective of (12). In the long run, the proposed LBUD auction is DSIC.

Proof: See Appendix B.

Remark 1: We need to be careful in interpreting the claim about the LBUD auction in Proposition 2, which focuses on the auction process itself for channel allocation of M-best channels. The entire spectrum access procedure also includes sensing and access after the auction. The resulting expected throughput and expected payoff of this secondary network are given in (1) and (20), respectively. Since the auction is designed based on per SU’s (instantaneous) payoff, there is no indication that the LBUD auction is DSIC for the overall expected payoff.

Remark 2: Using the secondary network expected payoff in (20) is desirable but difficult in design. The expectation is taken w.r.t. $T^*_j(n)$, the set of slots where SU $j$ can access channel $i$, in addition to the rate $R^*_j$ at those slots. It is random due to the learning and auction outcomes. Therefore, although maximizing the expected payoff function leads to an optimal strategy, the problem may not be tractable.

Remark 3: In our proposed LBUD auction, by letting each SU only reports its rates for the M-best channels for bidding, both channel availabilities and instantaneous rate are taken into account to maximize the throughput. Note that, instead of bidding estimated M-best channels by each SU, forming the set of $M$ channels with the highest $\tilde{\theta}^2(T^*_j(n))R^*_j(n)$ to bid has some disadvantages, despite it takes into account the (estimated) channel availability. The challenge involved in our problem stems from the unknown channel availability during bidding process. We use an underlying distributed learning process to learn the channel mean availability $\theta$ over time from sensing history, and use the auction approach to orthogonalize the channel assignment to SUs. The learning process would be order-optimal, if the choices of channels are from the estimated M-best channels, due to the proper trade-off between exploration-exploitation in learning (i.e., choose channels less sampled vs. most available) [3]–[5], [31]. If using the expected instantaneous payoff above, the auction procedure would interfere with the learning process. That is, SUs may choose channels not from estimated M-best channels to sense. Since the learning relies on sensing history, this would not grant the desired learning rate. The learning performance would in turn affect the auction outcome, leading to lower overall performance than our proposed mechanism. This has been demonstrated in our simulation studies (not shown).

C. The UD and the LBUD Auctions: Complexity vs. Overhead

In the UD auction mechanism, SUs bid for all $N$ primary channels. A total of $MN$ bids need to be sent to the auctioneer. This can result in a large communication overhead for large $N$. Unlike the UD auction, in the LBUD auction mechanism, each SU learns the primary channel occupation statistics and bids for the top $M$ channels that are considered by the SU to be the most available at the current time. Thus, bidding adaptively in the LBUD auction mechanism reduces communication overhead from $MN$ bids in UD auction to $M^2$ bids. The reduction can be particularly substantial in the broadband primary network environment where $M \ll N$. Thus, the LBUD auction mechanism not only improves the throughput performance when the primary channels have dissimilar availability statistics, but also reduces communication overhead.

In terms of the assignment complexity at the auctioneer, the adaptive price increment algorithm in Section IV-A improves the convergence speed of both the UD and the LBUD auctions. Between the two auction mechanisms, our simulations show that the LBUD auction mechanism takes more iterations to converge to the final channel assignment, as compared with that of the UD auction. To see why this is the case, note that the channels each SU can bid for are restricted to its estimated M-best channels. This increases the chance that SUs select the same channels during the iterative procedure. This may lead to a more likely event of having an overdemanded set (of channels), and as a result, slower convergence as compared with that for the UD auction.

V. Simulation Results

In this section, we present the simulation results to assess the performances of the adaptive pricing algorithm and our proposed LBUD auction mechanism. We assume $M$ SUs independently searching for idle channels among $N$ primary channels. In each time slot $n$, the channel availability state $X_i(n)$ for channel $i$ is independently drawn from Bernoulli distribution with mean $\theta_i$, unknown to SUs. We assume i.i.d Rayleigh fading for $h_i(n)$ of SU $j$’s link on channel $i$ over time and for different $i$ and $j$. The average received SNR over the secondary link, denoted as $\text{SNR} \triangleq \mathbb{E}[|h_i(n)|^2]/\sigma^2$, is set to be 8 dB. All simulations are performed for 50 Monte Carlo runs. We list all the case examples of the mean channel availability vector $\theta$ considered in our simulations in Table II. Cases 1 to 3 represent three different types of primary channel traffic loads for $N = 9$ and $M = 4$. Case 1 represents a scenario where the average
TABLE II  
SIMULATION CASES OF MEAN CHANNEL AVAILABILITY $\theta$

<table>
<thead>
<tr>
<th>$N$</th>
<th>$M$</th>
<th>Case</th>
<th>$\theta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>9</td>
<td>4</td>
<td>1</td>
<td>[0.3, 0.34, 0.5, 0.6, 0.67, 0.91, 0.2, 0.8, 0.7]</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2</td>
<td>[0.1, 0.2, ..., 0.9]</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3</td>
<td>[0.71, 0.72, ..., 0.79]</td>
</tr>
<tr>
<td>15</td>
<td>6</td>
<td>4</td>
<td>[0.3, 0.34, 0.5, 0.6, 0.67, 0.91, 0.2, 0.8, 0.7, 0.1, 0.45, 0.98, 0.56, 0.27, 0.43]</td>
</tr>
<tr>
<td></td>
<td></td>
<td>5</td>
<td>[0.1, 0.15, ..., 0.75, 0.8]</td>
</tr>
<tr>
<td></td>
<td></td>
<td>6</td>
<td>0.7 + 0.005 \times [2, 3, ..., 14, 16, 18]</td>
</tr>
</tbody>
</table>

Fig. 1. CDF of number of iterations under the UD auction ($N = 9$, $M = 4$; $\theta$: Case 1; SNR = 8 dB).

loads across different channels are random. Case 2 represents a case of dissimilar channels, where the loads are evenly spread out across channels. On the contrary, Case 3 shows a scenario where the average loads on all channels are similar. The similar examples for $N = 15$ and $M = 6$ are listed as Cases 4 to 6.

A. Adaptive Price Increment

We first show the improvement of the convergence speed using the adaptive price increment algorithm for $\Delta P_{n,l}$ proposed in Section IV-A (Algorithm 2). The channel availabilities are set randomly as Case 1. We set the average received SNR over each secondary link to be SNR = 8 dB. In Fig. 1, for the UD auction, we show the CDF of the number of iterations required to reach the final channel assignment under the adaptive price increment algorithm, as compared with that under the fixed baseline price increment in (14). The same comparison is also shown for the LBUD auction in Fig. 2. For both the UD and the LBUD auctions, we observe the substantial improvement of the convergence rate provided by the adaptive price increment algorithm, with the improvement being particularly pronounced in the LBUD auction. The adaptive price increment algorithm typically takes only a few iterations to reach the channel assignment solution. This thus leads to a significant reduction in complexity.

B. Impact of Learning and Exploiting Secondary Link Gain

1) Using Instantaneous Rate as Truthful Bidding: The bidding in both the UD and the LBUD auctions is a truthful bidding by using the instantaneous rate each SU has over its own secondary link. To study this effect on the performance, we consider a different bidding, i.e., sample mean bidding, where the bid that each SU submits is the estimated mean channel availability $\theta_i$ of each primary channel $i$. In Fig. 3, we compare the average payoff under the truthful bidding and sample mean bidding cases. The average payoff per SU at time slot $n$, denoted by $S(n)$, is given by

$$S(n) \triangleq \frac{1}{nM} \sum_{j=1}^{M} \sum_{i=1}^{N} \theta_i \mathbb{E} \left[ \sum_{k \in \mathcal{I}_j(n)} \left( R_j^i(k) - P_i(k) \right) \right].$$  \hspace{1cm} (20)

As it can be seen, for both the UD and the LBUD auctions, a substantial gain in payoff is achieved by bidding channels using the instantaneous secondary link channel gain rather than the sample mean availabilities of the primary channels. Additional gain is achieved by the LBUD auction as compared to the UD auction by further bidding only among the estimated $M$-best channels.

2) Multi-User Diversity Gain: We study the effect of instantaneous fading conditions of the secondary links on the performance under the LBUD auction mechanism. To demonstrate
we first consider an example where all the primary channels have similar load on average. Specifically, we set $N = 15$ and the mean channel availabilities as Case 6. Since $\theta_i$’s are similar, the primary channel is chosen would not be a factor that affects the throughput performance of an SU. However, the instantaneous fade of the secondary link over different channels does affect the throughput and is exploited in our proposed LBUD auction. This effect can be observed in Fig. 4, which shows the average throughput per SU for $M = 2, 4, 6, 8$. We see that the average throughput per SU increases as $M$ increases. The reason is that, in the LBUD auction, each SU will be assigned one of its estimated $M$-best channels. As $M$ increases, the SU can choose from more channels. Since the secondary fades over these channels are independent, the channel selection can take advantage from the instantaneous fade gains to capture a multi-user diversity gain; and this gain increases with $M$. Note that, since existing access policies do not consider instantaneous secondary fades, such a multi-user diversity gain cannot be achieved.

3) Gains From Multi-Channel Diversity vs. Using the $M$-Best Channels: In Fig. 5, we compare the average throughput per SU under the UD and the LBUD auctions, for Cases 1 to 3. From different cases of $\theta$ distributions, we see that, there is a trade-off between the gain of selecting channels among those that are less loaded and the gain of multi-channel diversity. In Cases 1 and 2, the average loads across channels are relatively spread out. Learning the mean availability of primary channels to avoid selecting those more loaded inferior channels is important to prevent throughput loss at SUs. Thus, the gain of selecting channels with higher mean channel availability outweighs the loss of multi-channel diversity due to only bidding among the $M$-best channels, and the LBUD auction outperforms the UD auction. In Case 3, the average loads are similar among channels, and choosing different channels will not impact the SUs’ throughputs. Instead, being able to choose from more channels will provide a more pronounced multi-channel diversity gain. The gain of choosing among the $M$-best channels diminishes, and the UD auction outperforms the LBUD auction due to the gain of multi-channel diversity. The experiments are repeated for Cases 4 to 6. The results are shown in Fig. 6, where the similar trade-offs are observed.

C. Comparison With Existing Access Policies

We further compare the performances of the UD and the LBUD auctions with that of the existing access policies. Specifically, the $\rho^{\text{RAND}}$ policy [4] and the DLF policy [5] are two existing decentralized access policies we are comparing with. Each policy implements the UCB1 algorithm as its underlying learning of the primary channel mean availabilities, and devises different mechanisms for channel selection and collision resolution. They are briefly described below:

- $\rho^{\text{RAND}}$ policy [4]: Each SU $j$ selects a random rank $r_j$ uniformly from 1 to $M$. It will then access the channel $i$ whose $I_j^i(n)$ is ranked $r_j^{th}$ in $\mathbf{V}(n)$. At time slot $n$, if a collision occurred in the previous slot, SU $j$ will re-draw $r_j$; Otherwise, it keeps the previously generated rank $r_j$ for channel selection.

- DLF policy [5]: At time slot $n$, SU $j$ selects the $r_j^{th}$-rank channel to access among the top $M$-ranked channels in terms of $\mathbf{V}(n)$, where the rank $r_j$ for each SU is generated in a round robin fashion $r_j = (j + n \mod M) + 1$.

The $\rho^{\text{RAND}}$ and the DLF policies are distributed with no central auctioneer, thus there are collisions but with less overhead. In addition, the channel selections of these two policies
only rely on mean channel statistics but do not utilize the instantaneous channel gains of the SUs.

In addition, for comparison, we consider the Bertsekas auction [25]. The Bertsekas auction provides a solution to the problem of \( M \) bidders (with unit demand) bidding among \( N \) objects. It is different from the UD auction as it does not consider the bidders’ incentives and thus is not dominant strategy incentive compatible. In addition, it cannot handle the primary network load condition.

For our comparison purpose, we provide a modified version of the Bertsekas auction which takes into account the primary channel condition. In the original Bertsekas auction, each SU is trying to find two channels with the highest and the second highest payoffs, respectively, among all channels. In the modified version, we let each SU find these two channels among its estimated \( M \)-best channels. Similar to the Bertsekas auction, this modified version is different from the LBUD auction as it does not consider the bidders’ incentives and therefore is not dominant strategy incentive compatible.

With the default setup parameters, Figs. 7–9 show the average throughput versus time slot \( n \) for the distribution of \( \theta \) in Cases 1 to 3, under the aforementioned access policies. In Fig. 10, we also compare the average throughput for these schemes with different mean channel availability \( \theta \), Case 4, where \( M = 6 \) and \( N = 15 \). As we see from these figures, both the LBUD and the UD auctions substantially outperform all the other access policies. The LBUD auction has the best performance for most cases except for the case when all channels have similar loads.

**VI. CONCLUSION**

In this paper, we considered the auction-based approaches for dynamic spectrum access with unknown primary channel availability statistics to the SUs. Assuming the primary channels are with distinct availability statistics, bidding such channels among SUs can be viewed as bidding multiple heterogeneous objects. We first applied the UD auction and explored the instantaneous link condition of each SU over the primary channels for its throughput maximization. To avoid accessing primary channels with high load, we further proposed the LBUD auction, in which distributed learning of the primary channels at each SU is performed and incorporated in the auction mechanism. The proposed LBUD auction explores both channel availability statistics and instantaneous link gains of the SUs in order to maximize SUs’ throughputs. It also reduces
communication overhead of the required bidding data over the UD auction. Such a joint consideration of both primary channel availabilities and secondary link conditions is not considered in existing works. Furthermore, considering the auction itself for channel allocation with the payoff defined, we showed that the proposed LBUD auction is DSIC. We further proposed an adaptive price increment algorithm to improve convergence speed of the iterative procedure in the auction. Numerical results show the effectiveness of our proposed auction mechanism in terms of the throughput gain.

APPENDIX A
PROOF OF PROPOSITION 1

Proof: We consider the UD auction procedure at time slot \( n \), as described in Section IV. The convergence of the iterative procedure under unit price increment, i.e., \( \Delta P_{n,1} = 1 \), has already been shown in [2] in the integer-valued bidding scenario. First consider the integer-valued scenario. Consider the \( l \)th and \((l + 1)\)th iterations under this unit price increment:

a) At Iteration \( l \): For SU \( j \), following Steps 3 and 4 of the UD auction, the auctioneer obtains the demand set \( D^l(P^{l}(n)) \) in (4), and the payoff difference \( \Delta \rho'(n, l) \) in (18). In Step 4b, assume that there exist overdemanded sets among the demand sets \( \{D^1(P^{l}(n)), \ldots, D^M(P^{l}(n))\} \). The auctioneer updates the price \( P^{l}_{k^1}(n) \) by (8), for \( i \in D^\text{min}(P^{l}(n)) \), i.e., those channels in the minimal overdemanded set.

Define \( S_{\text{min}} = \{ j : D^l(P^{l}(n)) = D^\text{min}(P^{l}(n)), \forall j \} \) as the set of SUs whose demanded set is the minimal overdemanded set. Note that since \( k^1_{1} \) in (16) is the channel with the highest current payoff for SU \( j \), we have \( k^1_{1} \in D^\text{min}(P^{l}(n)) \) if \( j \in S_{\text{min}} \). Assume \( \Delta \rho'(n, l) = L_j \geq 1 \). Let \( j^* \in S_{\text{min}} \) be the SU with

\[
L_{j^*} = \min_{j \in S_{\text{min}}} L_j.
\]

The corresponding price and the payoffs are updated as follows:

\[
P^{l+1}_{k^1_{1}}(n) = P^{l}_{k^1_{1}}(n) + 1,
\]

\[
\rho^{l}_{k^1_{1}}(n, l + 1) = \rho^{l}_{k^1_{1}}(n, l) - 1,
\]

\[
\rho^{l}_{k^2_{2}}(n, l + 1) = \rho^{l}_{k^2_{2}}(n, l),
\]

where (22) follows (8) with \( \Delta P_{n,1} = 1 \), and (23) is due to (15) and (22). For (24), since \( L_j \geq 1 \), we know \( k^2_{2} \notin D^\text{min}(P^{l}(n)) \).

b) At Iteration \( l+1 \): The same procedure in Steps 3 and 4 follows. From (23) and (24), we have the payoff difference as

\[
\rho^{l}_{k^1_{1}}(n, l + 1) - \rho^{l}_{k^2_{2}}(n, l + 1) = \rho^{l}_{k^1_{1}}(n, l) - \rho^{l}_{k^2_{2}}(n, l - 1)
\]

\[
= L_{j^*} - 1.
\]

In this case, channel \( k^1_{1} \) still remains the channel with the highest current payoff for the SU \( j^* \). As a consequence, the minimal overdemanded set does not change, and we have

\[
P^{l+2}_{k^1_{1}}(n) = P^{l+1}_{k^1_{1}}(n) + 1.
\]

The above procedure will be repeated \( L_{j^*} \) times till at the iteration \( l + L_{j^*} \), we have

\[
\rho^{l+L_{j^*}}_{k^1_{1}}(n, l + L_{j^*}) = \rho^{l}_{k^1_{1}}(n, l + L_{j^*}).
\]

Based on the definition of the demand set in (4), the above will lead to a change of the demand set \( D^{l+1}(P^{l+1}(n)) \) for SU \( j^* \), as compared to that in iteration \( l \). As a result, this may result in a change of the exclusive demanders \( B^E(D^{l+1}(P^{l+1}(n))) \) for \( D^{l+1}(P^{l+1}(n)) \), and consequently a change of the minimal overdemanded set \( D^\text{min}(P^{l+1}(n)) \).

Now, we apply our proposed adaptive price mechanism for SU \( j \) at time slot \( n \). At iteration \( l \), we assumed that \( \Delta \rho'(n, l) = L_j \), therefore, we have \( \Delta P_{n,l} = L \), where

\[
L = \min_{1 \leq j \leq M} L_j.
\]

Assume there are overdemanded sets among the demand sets \( \{D^{l}(P^{l}(n))\} \). The auctioneer updates the price \( P^{l}_{k^1_{1}}(n) \) in the price vector \( P^{l+1}(n) \), for \( i \in D^\text{min}(P^{l+1}(n)) \). Again, for \( j \in S_{\text{min}} \), we know that \( k^1_{1} \in D^\text{min}(P^{l+1}(n)) \). Find SU \( j^* \in S_{\text{min}} \), satisfying (21). We have two cases:

i) If \( L_{j^*} = L \): We have

\[
P^{l+1}_{k^1_{1}}(n) = P^{l}_{k^1_{1}}(n) + \Delta P_{n,l},
\]

\[
\rho^{l}_{k^1_{1}}(n, l + 1) = \rho^{l}_{k^1_{1}}(n, l + 1).
\]

Compare (26) and (29) with the unit price increment and adaptive price increment, respectively, we see that the latter reaches the same result of the former in just one iteration.

ii) If \( L_{j^*} > L \): We have

\[
\rho^{l}_{k^1_{1}}(n, l + [L_{j^*}/L]) \geq \rho^{l}_{k^1_{1}}(n, l + [L_{j^*}/L]),
\]

Compare (26) and (29) with the unit price increment and adaptive price increment, respectively, we see that the latter reaches the same result of the former faster in \( [L_{j^*}/L] \) iterations.

Since the convergence of the UD auction with the unit price increment has been shown [2], it follows that the UD auction with the adaptive price increment is also convergent.

The above analysis can be easily applied to the real-valued case. To see this, we note that any real-valued quantity can be converted to an integer value by multiplying it by an integer. Then, the iterative process follows with guaranteed convergence. The above proof can be straightforwardly applied to the LBUD auction to show the convergence under adaptive price increment. The only difference lies in the fact that in the LBUD auction, the demand set is considered as in (12).

APPENDIX B
PROOF OF PROPOSITION 2

Proof: To prove the proposition, we first convert the LBUD auction in the format of the UD auction. Then we show that, as the underlying learning of primary channels improves over time slot \( n \), the auction becomes DSIC.
In the LBUD auction, each SU $j$ observes its current valuation, i.e., $\bar{R}_i(n)$ for channel $i \in C_M(n)$, decides the corresponding bid $\bar{m}_i(n)$. The bids for each SU are submitted for its estimated $M$-best channels. Thus, the set of bids contains the information on the set of $M$ channels considered.

Let $C_M$ denote the set of $M$-best channels. The valuation of each channel for SU $j$ contains two facts: a) instantaneous rate $R_i(n)$ on channel $i$, and b) whether or not $i \in C_M$. Thus, bidding truthfully (i.e., a bid equals to the valuation) means truthfully selecting the $M$-best channels and truthfully reporting the instantaneous rate over each of these channels.

Now we design another UD auction. We construct a modified bid as follows:

$$\tilde{m}_i(n) = \begin{cases} m_i(n), & i \in C_i(n) \\ -\infty, & \text{otherwise} \end{cases}$$

(30)

where $C_i(n)$ denotes any set of $M$ channels in $C$. From above, we see that selecting a different set of $M$ channels is equivalent to setting different bid for each channel, resulting in a different set of bids. Based on (30), we have a corresponding modified valuation $\tilde{R}_i(n)$ of the channels as

$$\tilde{R}_i(n) = \begin{cases} R_i(n), & i \in C_M \\ -\infty, & \text{otherwise} \end{cases}$$

(31)

In the LBUD auction, each SU $j$ has its own estimated $M$-best channel $C_M(n)$, thus its bid is given by

$$\tilde{R}_j(n) = \begin{cases} R_j(n), & i \in C_M(n) \\ -\infty, & \text{otherwise} \end{cases}$$

(32)

Each SU $j$ uses this modified bid $\tilde{R}_j(n)$ to bid for every channel $i \in C$. At the auctioneer, the demand set for each SU $j$ is denoted as $D_j((P_j(n)))$, which is similar to (4) and is given by

$$D_j((P_j(n))) = \arg \max_{i \in C} \left( \tilde{R}_i(n) - P_j(n) \right).$$

(33)

The auctioneer will carry out the assignment using the UD auction mechanism described in Section III. Using (32), we examine (12) and (33). We can see that

$$D_j((P_j(n))) = D_M((P_j(n))).$$

(34)

As mentioned in the main text, the distributed learning of $M$-best channels by (10) and (11) under the UCB1 algorithm has been shown to be order-optimal in terms of the learning rate over time [31]. Specifically, as the time slot $n \to \infty$, the probability of not selecting the true $M$-best channel is

$$\text{Prob}(C_M(n) \neq C_M) = O \left( \frac{\log n}{n} \right) \to 0.$$

Thus, we have $\tilde{R}_j(n) \to \tilde{R}_j(n)$ in probability. In other words, the bids for each SU $j$ converges to the valuations of $M$-best channels in probability. Consequently, let

$$\tilde{D}_j((P_j(n))) = \left\{ \arg \max_{i \in C} \left( \tilde{R}_i(n) - P_j(n) \right) \right\}.$$

From (34), we have $D_M((P_j(n))) \to \tilde{D}_j((P_j(n)))$ in probability.

Thus, in the long run as $n \to \infty$, the LBUD auction is essentially equivalent to the new UD auction we constructed. Since the UD auction is dominant strategy incentive compatible, it follows that the proposed LBUD also carries such a property.

\section*{REFERENCES}


Marjan Zandi (S’13) received the B.S. degree in electrical engineering from Sharif University of Technology, Tehran, Iran, in 1994. After her graduation, she worked as a Hardware Design Engineer in telecommunication industry. She received the Master’s degree in information technology security from University of Ontario Institute of Technology, Oshawa, Canada, in 2007 and the Ph.D. degree in electrical engineering from University of Ontario Institute of Technology, Oshawa, Canada, in 2014. Her research interests are in the areas of statistical signal processing, wireless communications, and communication networks.

Min Dong (S’00–M’05–SM’09) received the B.Eng. degree from Tsinghua University, Beijing, China, in 1998, and the Ph.D. degree in electrical and computer engineering with minor in applied mathematics from Cornell University, Ithaca, NY, USA, in 2004. From 2004 to 2008, she was with Corporate Research and Development, Qualcomm Inc., San Diego, CA, USA. In 2008, she joined the Department of Electrical Computer and Software Engineering at University of Ontario Institute of Technology, Ontario, Canada, where she is currently an Associate Professor. She also holds a status-only Associate Professor appointment with the Department of Electrical and Computer Engineering, University of Toronto since 2009. Her research interests are in the areas of statistical signal processing for communication networks, cooperative communications and networking techniques, and stochastic network optimization in dynamic networks and systems.

Dr. Dong received the Early Researcher Award from Ontario Ministry of Research and Innovation in 2012, the Best Paper Award at IEEE ICC in 2012, and the 2004 IEEE Signal Processing Society Best Paper Award. She has served as an Associate Editor for the IEEE TRANSACTIONS ON SIGNAL PROCESSING from 2010 to 2014, and an Associate Editor for the IEEE SIGNAL PROCESSING LETTERS from 2009 to 2013. She has been an elected member of IEEE Signal Processing Society Signal Processing for Communications and Networking (SP-COM) Technical Committee since 2013.

Ali Grami (M’86–SM’06) received the B.Sc., M.Eng., and Ph.D. degrees from the University of Manitoba, McGill University, and the University of Toronto, respectively, all in electrical engineering. Following his graduation, he joined Nortel Networks, where he was involved in the research, design, and development of North America’s first digital cellular mobile system. He then joined Telesat Canada, where he was the lead researcher and principal designer of Canada’s Anik-F2 Ka-band system, the world’s first broadband access satellite systems and the first satellite to successfully commercialize the Ka-band technology. While he was with the industry, he taught at the University of Ottawa and Concordia University, and received the United Nations TOKTEN award. In 2003, Dr. Grami, as a founding faculty member, joined the University of Ontario Institute of Technology (UOIT), where he has led the development of B.Eng., M.Eng., and Ph.D. programs in electrical and computer engineering. He is the author of the undergraduate-level textbook Introduction to Digital Communications (Elsevier, 2015).