A Comparison between Latency Insertion Method and Relaxation Method in Transient Thermal Analysis

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Introduction
Rapid advancement of semiconductor industry in recent years has improve dramatically the power of IC.

Problems:
Recent trends in scaling down conventional transistor size and three-dimensional stacked ICs caused the larger power consumption per unit area, so as the amount of Joule heat generated. It becomes more difficult to dissipate the heat produced in electrical circuits.

Understanding transient thermal effects in electrical circuits is critical in designing stage.

Thermal Equivalent Circuit Network

Fourier’s Law
\[ q = -k \nabla T \] (1)

Applying eq (1) to thermal cell
\[ Q = \frac{-T_i - T_j}{\Delta x} \] (2)

Heat resistance \( R \)
\[ R = \frac{\Delta x}{kS} \] (3)

\[ Q = \frac{1}{R} [T_i - T_j] \] (4)

Thermal Conduction
Equation gained from Fourier’s Law
\[ Q = \frac{1}{R} [T_i - T_j] \] (4)

Heat capacitance
\[ C = \varepsilon_r \rho \Delta x \] (5)

Ohm’s Law
\[ V = IR \] (6)

Capacitance
\[ C = \frac{I}{\Delta V} \] (7)

Latency Insertion Method

Branch structure

Node structure

Current updating formulation
\[ I_i^{n+1} = 2I_i^{n} - R_A I_i^{n} + \frac{V_i^{n+1} - V_i^{n-1}}{2\Delta t} \] (8)

Voltage updating formulation
\[ V_i^{n+1} = V_i^{n} + \frac{E_i^{n+1} - E_i^{n}}{\Delta t} \] (9)

Stability condition (time-step size)
\[ \Delta t \leq \sqrt{\frac{2R_A}{N_i}} \] (10)
Relaxation Method

Gauss-Seidel Method

- Nodal Analysis Method

\[ GV = I \] (11)

- Gauss-Seidel Method's updating

\[ V_{i}^{(n+1)} = \frac{1}{G_{i}} \left( I_{i} - \sum_{j \neq i}^{N} G_{ij} V_{j}^{(n+1)} - \sum_{j \neq i}^{N} G_{ij} V_{j}^{(n)} \right) \] (12)

Convergence condition

\[ \sqrt{\sum_{i}^{N} (V_{i}^{(n+1)} - V_{i}^{(n)})^2} < \epsilon_{\text{max}} \] (3)

- Conductance

\[ \frac{1}{R_{i}} = \frac{1}{R_{\text{amb}}}, R_{\text{amb}} = \frac{1}{k} \cdot \frac{1}{S_{\text{amb}}} \] (14)

Example Validation

Fig 3. 2D cross-section model of IC

Fig 6. Equivalent Circuit

Fig 7. Transient thermal results at point E

Table 1. Comparison of computing time

<table>
<thead>
<tr>
<th>Condition</th>
<th>LIM</th>
<th>Gauss-Seidel Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stability condition</td>
<td>(10^{-5})</td>
<td>(10^{-5})</td>
</tr>
<tr>
<td>(\Delta t)</td>
<td>(2.87 \times 10^{-5})</td>
<td>(2.87 \times 10^{-5})</td>
</tr>
<tr>
<td>CPU Times (s)</td>
<td>4.78</td>
<td>54.31</td>
</tr>
</tbody>
</table>

Example Validation

- Heat source: \( V_{\text{heat}} \)

- Heat transfer coefficient \( C_{\text{heat}} \)

- Thermal resistance \( R_{\text{amb}} \)

- Thermal resistance to nonturied boundary cell \( R_{\text{amb}} = R_{S} + R_{\text{amb}} \) (3)

Conclusion

In summary, a comparison between LIM and Relaxation Method in Transient Thermal Analysis was conducted in this research.

- From the transient thermal results, LIM performed thermal analysis at a faster speed than relaxation method when the results from both of methods matched.

- By changing convergence conditions and time-step sizes, Gauss-Seidel method can be accelerated, though this may lead to a great sacrifice of precision.